Radiation field in a superstrong magnetoactive electron plasma

Chih-Kang Chou*
Institute of Astronomy and Department of Physics, National Central University, Chungli, Taiwan, Republic of China
(Received 26 March 1999)

Using the language of quantum field theory, we present a concise derivation for the electromagnetic vector potential A_μ, which is valid for an anisotropic superstrong magnetoactive electron plasma. It is shown that the expression for the vector potential A_μ can be reduced to various known limits. Applications to important problems in astrophysics are briefly discussed. The relevance of our result to the recent development of the collective interaction between intense neutrino fluxes and stellar plasmas is briefly stressed.
[S1063-651X(99)09311-3]

PACS number(s): 52.25.Jm, 51.60.+a, 52.35.Mw, 98.62.En

I. INTRODUCTION

The role of strong magnetic field in high-energy astrophysics has attracted considerable theoretical interest ever since the discovery of pulsars. It is now generally believed that collapsed stellar objects such as dwarf stars and neutron stars are very likely to possess strong magnetic fields of the order of 10^8-10^{12} G a million times greater than laboratory-produced magnetic field.

The origin of ultrastrong magnetic fields in astrophysics is usually attributed to flux conservation during gravitational collapse of magnetic stars [1]. Alternatively, intense magnetic fields may be maintained in the interiors of neutron stars either by Landau orbital ferromagnetism of the degerrate electrons [2] or by neutron ferromagnetism [2–5]. Magnetic fields of the order of 10^{12} G could be maintained in the interiors of neutron stars if Landau orbital ferromagnetism of the electrons operates, whereas interior fields of the order of 10^{15} G or greater may be generated if there is neutron ferromagnetism.

Primordial cosmic magnetic fields produced by dynamo mechanism in vector inflationary scenarios has been emphasized by Lewis [6] and Harrison [7]. Several alternative mechanisms for primordial magnetic fields are discussed by Opher and Wichoski [8]. More recently Shukla *et al.* [9] have shown that the ponderomotive force of a nonuniform intense neutrino beam can generate large-scale quasistationary magnetic fields in a dense electron plasma. This mechanism can be responsible for the origin of magnetic fields in the early universe. Whereas the origin of these fields is still unclear and more or less speculative, we can contemplate studying the roles of these fields they play in high-energy astrophysics.

Neutron stars are very quiet stars when compared with ordinary stars where thermal nuclear reactions still take place; they are very cold, probably among the coldest places in the universe. The interior of the star is now believed to be a two-sphere solid configuration separated by a lubricant composed of superfluid neutrons. In the inner sphere, very few electrons exist and matter is predominantly composed of interacting baryons. In the outer sphere (the crust) where density is relatively low, matter is primarily composed of a partially ionized plasma immersed in a superstrong magnetic field. Thus, for the crust two main effects can be discerned. One is related to the electrons treated as an independent magnetoactive plasma, and the second has to do with the atoms that constitute the bulk of the surface of the crust.

Atomic physics in ultrastrong magnetic fields was pursued by Ruderman [10] and Spruch *et al.* [11]. The effects of the plasma have been considered to some extent by Canuto and Chou [12] by using an anisotropic pressure tensor derived from the Canuto-Chiu [13] equation of state for an electron gas in a superstrong magnetic field.

Among the processes that are of primary importance for the thermal history of a neutron star are electron-positron annihilation into neutrinos and photoneutrinos. The effects of a strong magnetic field on these processes have been studied by many authors [14–16]. More recently, the effect of superstrong magnetic field on electron capture has been emphasized by Dai, Lu, and Peng [17].

In order to consider the effects of the magnetoactive electron plasma on the emission of photoneutrinos, it is necessary to know the proper form of the vector potential A_μ, which is appropriate in the presence of the plasma and the magnetic field. The correct form of A_μ has been given by the author and extensively used by many authors [18–20]. Although the derivation of the proper form of A_μ has previously been given by Adams, Ruderman, and Woo [21] for an isotropic plasma in the absence of external magnetic fields, and discussed by Melrose [22] for anisotropic magnetoactive plasma, however, it seems to the best of my knowledge that the correct formulation for this simple but important problem has never been published in the open literature. The role of a strongly magnetoactive electron plasma is clearly enhanced in view of the very recent theoretical interest to study the collective interactions between neutrinos and background stellar plasmas [23–27]. The coupling between neutrinos and plasmons occurs due to the weak Fermi interaction force [28] involving neutrinos, bosons, and plasma electrons. More specifically, intense neutrino fluxes propagating through the plasma perturb the electron number density through the Fermi interaction and thereby produce upper and lower energy-level neutrinos. The latter interact with the original neutrino flux to produce a low-frequency ponderomotive force [25], which reinforces the density perturbations

*Electronic address: Chou@joule.phy.ncu.edu.tw

in the background plasma. Consequently, the original neutrino energy density is depleted and transferred to plasma waves which, in turn, heat the plasma electrons via Landau damping.

Since there exist superstrong magnetic fields of the order of several billion gauss at neutron star surface, it is necessary [26,27] to include the ambient magnetic fields in the theory of collective interactions between neutrinos and stellar matter. The inclusion of homogeneous magnetic fields leads to such parametric excitations as whistlers and X-mode radiation that can account for the ponderomotive force of the intense neutrino fluxes. However, in all these investigations, the background magnetoactive plasma has always been treated by the standard method of classical plasma physics. It is well known that for superstrong magnetic fields the motion of the plasma electrons are quantized into Landau orbits [13] so that the background plasma should be treated as a magnetized electron gas by the methods of quantum mechanics. It is our hope to extend the theory of nonlinear interactions between neutrinos and stellar matter recently developed by Bethe [28], Bingham [24,25], and Shukla *et al.* [26,27] such that a full quantum-mechanical treatment is applied to both the incident intense neutrino fluxes and the stellar plasma immersed in a superstrong magnetic field. More specifically, the theory developed by Shukla *et al.* [9,26,27] and Bingham *et al.* [24,25] for the nonlinear coupling between intense neutrino fluxes and a dense magnetized plasma is a semiclassical theory in that the incident neutrino fluxes are governed by a nonlinear Klein-Gordon equation in relativistic quantum mechanics. On the other hand. The variety of magnetized plasma waves that are driven by the neutrino energy density are studied in the general framework of standard plasma physics.

As previously emphasized, we are now in the process of formulating a full quantum-theoretical treatment for the nonlinear interaction between neutrinos and stellar plasmas immersed in superstrong magnetic fields. As a first step in our endeavour, we consider the radiation field A_μ which is appropriate to depict the magnetoactive stellar plasma and show that it can be reduced to various known limits. We shall choose the gauge so that the scalar potential ($A_4 = i\phi$) vanishes. We then quantize A_μ in terms of the Maxwell operator Λ_{ij} [as defined by Eq. (26) of Sec. II] and the annihilation and creation operators. Applications of our result to important astrophysical problems such as the neutiono energy-lose rates due to Compton scattering in a relativistic magnetoactive plasma and the rich variety of plasma excitations driven by the nonlinear coupling between the neutrino fluxes and the stellar plasma previously mentioned will be presented in future publications.

II. PLASMA ENERGY DENSITY

Collective excitations in a plasma may be initiated by the energy supply from an external source. The energy of a plasma wave appears in the electromagnetic field and in the charged-particle kinetic energy which is associated with the coherent wave motions. If the external supply of electromagnetic energy to the plasma is cut off, the absorption which is always present, no matter how small, will ultimately convert the energy of the plasma into heat.

We turn now to consider the energy density for the plasma. Maxwell's equations may be written in the form

$$\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}, \quad \vec{\nabla}\cdot\vec{D} = 4\pi\rho_{\text{ext}}, \quad \vec{\nabla}\cdot\vec{B} = 0,$$

$$\nabla\times\vec{B} = \frac{4\pi}{c}\vec{J}_{\text{ext}} + \frac{1}{c}\frac{\partial\vec{D}}{\partial t}, \quad \vec{D} = \vec{\vec{\varepsilon}}\cdot\vec{E} = \vec{E} + 4\pi\vec{P}, \tag{1}$$

$$\frac{\partial\vec{D}}{\partial t} = 4\pi\vec{J}_{\text{ind}} + \frac{\partial\vec{E}}{\partial t}, \quad \vec{J}_{\text{ind}} = \frac{\partial\vec{P}}{\partial t}.$$

In the above equation, $\partial\vec{D}/\partial t$ is the sum of the displacement current (caused by the change in the electric field) and the internally induced current $\vec{J}_{\text{ind}}$, whereas $\vec{J}_{\text{ext}}$ represents the current density externally applied. With this form of the Maxwell's equations we then have

$$\frac{\partial}{\partial t}\left(\frac{\vec{E}\cdot\vec{D} + \vec{B}\cdot\vec{B}}{8\pi}\right) = -\vec{\nabla}\cdot\left(\frac{c}{4\pi}\vec{E}\times\vec{B}\right) - \vec{J}_{\text{ext}}\cdot\vec{E}$$

or

$$\frac{\partial W}{\partial t} + \vec{\nabla}\cdot\vec{S} = -\vec{J}_{\text{ext}}\cdot\vec{E}. \tag{2}$$

If there is no external current,

$$\frac{\partial W}{\partial t} = -\vec{\nabla}\cdot\vec{S} = \frac{1}{4\pi}\left(\vec{B}\cdot\frac{\partial\vec{B}}{\partial t} + \vec{E}\cdot\frac{\partial\vec{D}}{\partial t}\right). \tag{3}$$

We note that although Eqs. (2) and (3) are rigorous at every instant of time, our concern will be only in the average behavior of these equations over a period of time since we are not interested in transient effects. However, for the purpose of studying wave motion in a plasma it is convenient to take the fields $\vec{E}$, $\vec{B}$, and $\vec{D}$ to be complex and then substitute for these fields their corresponding real values. Thus in Eq. (3) we replace $\vec{E}$ and $\vec{D}$ by $\frac{1}{2}(\vec{E}+\vec{E}^*)$ and $\frac{1}{2}(\vec{D}+\vec{D}^*)$, respectively, and similarly for $\vec{B}$. The average energy density can now be rewritten in the form

$$\langle W\rangle = \frac{1}{16\pi}\int dt\{\vec{B}\cdot\dot{\vec{B}}^* + \vec{B}^*\cdot\dot{\vec{B}} + \vec{E}\cdot\dot{\vec{D}}^* + \vec{E}^*\cdot\dot{\vec{D}}\}, \tag{4}$$

where the products $\vec{E}\cdot\vec{D}$ and $\vec{B}^*\cdot\vec{B}^*$, etc., have been dropped from Eq. (4) since their time average vanishes.

If the fields $\vec{E}$ and $\vec{B}$ are further assumed to be strictly monochromatic, i.e.,

$$\vec{E} = \vec{E}_0 e^{-i\omega t}, \quad \vec{B} = \vec{B}_0 e^{-i\omega t}$$

with $\vec{E}_0$ and $\vec{B}_0$ independent of time, then

$$\vec{E}^*\cdot\vec{E} = |\vec{E}_0|^2 = 2\langle(\text{Re}\,\vec{E})^2\rangle, \quad \vec{B}^*\cdot\vec{B} = 2\langle(\text{Re}\,\vec{B})^2\rangle$$

and the energy density becomes

$$\langle W\rangle = \frac{1}{8\pi}\{\langle(\mathrm{Re}\,\vec{B})^2\rangle + \langle(\mathrm{Re}\,\vec{E})^2\rangle\}, \tag{5}$$

which is the usual expression for the electromagnetic energy density.

In a dispersive medium such as the electron plasma (with and without external magnetic field), the situation is much more complicated. The presence of arbitrary dispersion gives rise in general to energy dissipation, i.e., a dispersive medium is also an absorbing medium. Thus, in a dispersive medium, energy must be supplied to maintain the collective excitations and to overcome the dissipasive losses into heat. The building-up of the waves and the dissipative losses can be represented by fields with a complex frequency. Before evaluating the time average of Eq. (3) for this case, let us note that for monochromatic plane waves one has

$$\begin{aligned}\langle\vec{A}\cdot\vec{B}\rangle &= \langle \mathrm{Re}(\vec{A}_0 e^{-i\varphi})\cdot \mathrm{Re}(\vec{B}_0 e^{-i\varphi})\rangle \\ &= \tfrac{1}{4}(\vec{A}_0\cdot\vec{B}_0^* + \vec{A}_0^*\cdot\vec{B}_0)\exp(2\omega_i t)\end{aligned} \tag{6}$$

with $\varphi=\omega t$, $\omega=\omega_r+i\omega_i$, $\omega_r=\mathrm{Re}\,\omega$, $\omega_i=\mathrm{Im}\,\omega$.

The rate of change of the energy density may now be calculated by applying Eq. (6) to Eq. (3). To calculate $\langle\dot{W}_E\rangle$ we note that

$$\begin{aligned}\langle\dot{W}_E\rangle &= \left\langle\frac{\partial W_E}{\partial t}\right\rangle = \frac{1}{4\pi}\left\langle\vec{E}\cdot\frac{\partial\vec{D}}{\partial t}\right\rangle \\ &= \tfrac{1}{4}\{\omega^*\vec{E}_0\cdot\vec{D}_0^* - \omega\vec{E}_0^*\cdot\vec{D}_0\}e^{-i(\varphi-\varphi^*)} \\ &= \tfrac{1}{4}\{\omega_r\vec{E}_0^*\cdot(\overleftrightarrow{\varepsilon}^+ - \overleftrightarrow{\varepsilon})\cdot\vec{E}_0 \\ &\quad - i\omega_i\vec{E}_0^*\cdot(\overleftrightarrow{\varepsilon}^+ + \overleftrightarrow{\varepsilon})\cdot\vec{E}_0\}e^{2\omega_i t},\end{aligned} \tag{7}$$

where $\overleftrightarrow{\varepsilon}^+$ is the Hermitian conjugate of $\overleftrightarrow{\varepsilon}$,

$$\varepsilon_{ij}^+ = (\tilde{\varepsilon}_{ij})^* = \varepsilon_{ji}^*,$$

that is, the complex conjugate of the transposed matrix, and where we have used the relation

$$\vec{E}_0\cdot\overleftrightarrow{\varepsilon}^*\cdot\vec{E}_0^* = \vec{E}_0^*\cdot\overleftrightarrow{\varepsilon}^+\cdot\vec{E}_0.$$

Similarly, from Eq. (3) we have

$$\begin{aligned}\left\langle\frac{\partial W_B}{\partial t}\right\rangle &= \left\langle\vec{B}\cdot\frac{\partial\vec{B}}{\partial t}\right\rangle = \tfrac{1}{4}\vec{B}_0^*\cdot\vec{B}_0(\omega^*-\omega)\exp[-i(\varphi-\varphi^*)] \\ &= \tfrac{1}{4}\{2\omega_i\vec{B}_0^*\cdot\vec{B}_0\exp(2\omega_i t)\}\end{aligned} \tag{8}$$

so that the time-averaged total energy density becomes

$$\begin{aligned}\left\langle\frac{\partial W}{\partial t}\right\rangle &= \frac{1}{16\pi}\{2\omega_i\vec{B}_0^*\cdot\vec{B}_0 + \omega_i\vec{E}_0^*\cdot(\overleftrightarrow{\varepsilon}^+ + \overleftrightarrow{\varepsilon})\cdot\vec{E}_0 \\ &\quad + i\omega_r\vec{E}_0^*\cdot(\overleftrightarrow{\varepsilon}^+ - \overleftrightarrow{\varepsilon})\cdot\vec{E}_0\}e^{2\omega_i t}.\end{aligned} \tag{9}$$

The above equation represents the rate at which energy must be supplied to build up the collective excitations and to overcome the dissipative losses into heat. For a lossless plasma in the steady state $\partial W/\partial t=0, \omega_i=0$. It follows from Eq. (9) that the necessary and sufficient condition for a loss-free plasma is that the dielectric tensor must be Hermitian, namely, $\overleftrightarrow{\varepsilon}^+=\overleftrightarrow{\varepsilon}$. On the other hand, if $\omega_i=0$, and the dielectric tensor is not Hermitian $\overleftrightarrow{\varepsilon}^+\neq\overleftrightarrow{\varepsilon}$, then $\langle\partial W/\partial t\rangle$ gives directly the heat loss rate, i.e.,

$$\begin{aligned}Q = \left\langle\frac{\partial W}{\partial t}\right\rangle_{\omega_i=0} &= i\omega_r\vec{E}_0^*\cdot(\overleftrightarrow{\varepsilon}^+ - \overleftrightarrow{\varepsilon})\cdot\vec{E}_0/16\pi \\ &= i\omega_r\vec{E}_0^*\cdot\overleftrightarrow{\varepsilon}_a\cdot\vec{E}_0/8\pi,\end{aligned} \tag{10}$$

where $\overleftrightarrow{\varepsilon}_a$ is the anti-Hermitian part of the dielectric tensor.

For the simplest case of an isotropic dispersive medium in which the dielectric tensor is a scalar $\varepsilon_{ij}=\varepsilon\delta_{ij}$, Eq. (10) reduces to

$$Q = i\omega_r\vec{E}_0^*\cdot\vec{E}_0(-i\,\mathrm{Im}\,\varepsilon)/8\pi = \langle(\mathrm{Re}\,\vec{E})^2\rangle\omega_r\,\mathrm{Im}\,\varepsilon/4\pi. \tag{11}$$

Equation (11) shows that the dissipation (absorption) of energy is determined by the imaginary part of ε. According to the law of increase of entropy, the sign of these heat losses is determinate, i.e., $Q>0$, and consequently

$$Q>0, \quad \mathrm{Im}\,\varepsilon>0. \tag{12}$$

In a loss-free plasma, $\overleftrightarrow{\varepsilon}^+=\overleftrightarrow{\varepsilon}$, the averaged electrostatic energy density may be written as

$$\begin{aligned}W_E &= \int dt\left\langle\frac{\partial W_E}{\partial t}\right\rangle = \frac{1}{16\pi}\int dt\,\varepsilon_{ij}\left[E_j\frac{\partial}{\partial t}E_i^* + E_i^*\frac{\partial}{\partial t}E_j\right] \\ &= \frac{1}{16\pi}\varepsilon_{ij}\int dt\frac{\partial}{\partial t}(E_i^*E_j) = \frac{1}{8\pi}\varepsilon_{ij}\langle\mathrm{Re}(E_i^*E_j)\rangle\end{aligned} \tag{13}$$

provided the field $\vec{E}$ is monochromatic.

It is easy to see that even for the simplest case of an isotropic electron plasma at zero temperature for which $\varepsilon_{ij}=(1-\omega_p^2/\omega^2)\delta_{ij}$, Eq. (13) can become negative; in fact,

$$W_E = \frac{1}{8\pi}\left(1-\frac{\omega_p^2}{\omega^2}\right)\langle(\mathrm{Re}\,\vec{E})^2\rangle<0 \quad \text{for} \quad \omega<\omega_p. \tag{14}$$

This unphysical result is due to the fact that so far we have considered only monochromatic waves of a single frequency. As was emphasized by Landau, for strictly monochromatic fields there is no steady accumulation of electromagnetic energy. To remove this difficulty we therefore consider non-strictly monochromatic fields with frequencies in a narrow range about the mean value ω_0 of the carrier. Taking the Fourier transform of $\vec{E}(t)$ we have

$$\vec{E}(t) = \int_{-\infty}^{\infty} d\omega\,\vec{E}(\omega)\exp(-i\omega t), \tag{15}$$

where $\vec{E}(\omega)$ is assumed to have a sharp maximum at $\omega=\omega_0$.

Equation (15) may be rewritten in a more transparent form by performing a simple change of variable,

$$\vec{E}(t)=e^{-i\omega_0 t}\int_{-\infty}^{\infty}d\alpha\,\vec{E}(\omega_0+\alpha)e^{-i\alpha t}=e^{-i\omega_0 t}\vec{E}_0(t). \tag{16}$$

The field amplitude in Eq. (16) is now a slowly varying function of time. Similarly the field D takes the form

$$D(t)=\int_{-\infty}^{\infty}d\omega\,\overleftrightarrow{\varepsilon}(\omega)\cdot\vec{E}(\omega)e^{-i\omega t}$$
$$=e^{-i\omega_0 t}\int_{-\infty}^{\infty}d\alpha\,\overleftrightarrow{\varepsilon}(\omega_0+\alpha)\cdot\vec{E}(\omega_0+\alpha)e^{-i\alpha t}. \tag{17}$$

Since $\vec{E}_0(t)$ is assumed to be a slowly varying function of time so that $\alpha\ll\omega_0$, we can expand the dielectric tensor in a power series of α and retain the first term only,

$$\vec{D}(t)=e^{-i\omega_0 t}\int_{-\infty}^{\infty}d\alpha\left[\overleftrightarrow{\varepsilon}(\omega_0)+\alpha\left.\frac{\partial\overleftrightarrow{\varepsilon}}{\partial\omega}\right|_{\omega=\omega_0}+\cdots\right]\cdot\vec{E}(\omega_0+\alpha)e^{-i\alpha t}$$
$$=\overleftrightarrow{\varepsilon}(\omega_0)\cdot e^{-i\omega_0 t}\vec{E}_0(t)+e^{-i\omega_0 t}\left.\frac{\partial\overleftrightarrow{\varepsilon}}{\partial\omega}\right|_{\omega=\omega_0}\cdot\int_{-\infty}^{\infty}\alpha\,d\alpha\,e^{-i\alpha t}\vec{E}(\omega+\alpha)$$
$$=\overleftrightarrow{\varepsilon}(\omega_0)\cdot\vec{E}(t)+ie^{-i\omega_0 t}\left.\frac{\partial\overleftrightarrow{\varepsilon}}{\partial\omega}\right|_{\omega=\omega_0}\cdot\frac{\partial\vec{E}_0(t)}{\partial t}, \tag{18}$$

where we have used

$$\frac{\partial}{\partial t}\vec{E}_0(t)=\frac{\partial}{\partial t}\int_{-\infty}^{\infty}d\alpha\,e^{-i\alpha t}\vec{E}(\omega_0+\alpha)$$
$$=-i\int_{-\infty}^{\infty}\alpha\,d\alpha\,e^{-i\alpha t}\vec{E}(\omega_0+\alpha).$$

Differentiation of Eq. (18) with respect to time then yields

$$\frac{\partial\vec{D}}{\partial t}=\overleftrightarrow{\varepsilon}(\omega_0)\cdot\left\{-i\omega_0\vec{E}(t)+e^{-i\omega_0 t}\frac{\partial\vec{E}_0(t)}{\partial t}\right\}$$
$$+i\left.\frac{\partial\overleftrightarrow{\varepsilon}}{\partial\omega}\right|_{\omega=\omega_0}\cdot\left[-i\omega_0\frac{\partial\vec{E}_0(t)}{\partial t}+\frac{\partial^2}{\partial t^2}\vec{E}_0(t)\right]e^{-i\omega_0 t}$$
$$=-i\omega_0\overleftrightarrow{\varepsilon}(\omega_0)\cdot\vec{E}+\left[\frac{\partial}{\partial\omega}[\omega\overleftrightarrow{\varepsilon}(\omega)]\right]_{\omega=\omega_0}\cdot e^{-i\omega_0 t}\frac{\partial\vec{E}_0(t)}{\partial t}, \tag{19}$$

where we have neglected the term $\partial^2\vec{E}_0(t)/\partial t^2$ which is second order in the small frequency α. Using Eqs. (16)–(19) in Eq. (4) we obtain the following expression for the electrical part of the electromagnetic energy density:

$$\langle W_E\rangle=\frac{1}{16\pi}\int dt\left\{\vec{E}\cdot\left[i\omega\overleftrightarrow{\varepsilon}^{+}(\omega)\cdot\vec{E}^*+\frac{\partial}{\partial\omega}(\omega\overleftrightarrow{\varepsilon}^{+})\cdot e^{i\omega t}\frac{\partial\vec{E}_0^*}{\partial t}\right]+\vec{E}^*\cdot\left[-i\omega\overleftrightarrow{\varepsilon}\cdot\vec{E}+\frac{\partial}{\partial\omega}(\omega\overleftrightarrow{\varepsilon})\cdot e^{-i\omega t}\frac{\partial\vec{E}_0}{\partial t}\right]\right\}, \tag{20}$$

where the subscript in the mean frequency ω_0 has been dropped.

For a loss-free plasma, the dielectric tensor is Hermitian so that

$$\overleftrightarrow{\varepsilon}^{+}=\overleftrightarrow{\varepsilon}$$

and

$$\vec{E}\cdot\overleftrightarrow{\varepsilon}^{+}\cdot\vec{E}^*=\vec{E}^*\cdot\overleftrightarrow{\varepsilon}\cdot\vec{E}.$$

Equation (20) then reduces to

$$\langle W_E\rangle=\frac{1}{16\pi}\frac{\partial}{\partial\omega}(\omega\varepsilon_{ij})\int dt\,\frac{\partial}{\partial t}(E_{0i}^*E_{0j})$$
$$=\frac{1}{16\pi}\frac{\partial(\omega\varepsilon_{ij})}{\partial\omega}(E_{0i}^*E_{0j}), \tag{21}$$

where we have used Eq. (16). Similarly, the magnetic part of the total energy density may be shown to be

$$\langle W_B \rangle = \frac{1}{16\pi} B_{0i}^* B_{0i} . \tag{22}$$

It is to be noted that the fields $\vec{E}_0, \vec{B}_0$ are now slowly varying functions of time. The total electromagnetic energy density is therefore given by

$$\langle W \rangle = \frac{1}{16\pi} \left[B_i^* B_i + E_i^* \frac{\partial(\omega \varepsilon_{ij})}{\partial \omega} E_j \right]. \tag{23}$$

From Maxwell's equations we readily obtain

$$\vec{B} = \frac{c}{\omega} \vec{k} \times \vec{E}, \quad \vec{k} \times \vec{B} = -\frac{\omega}{c} \overleftrightarrow{\varepsilon} \cdot \vec{E},$$

so that

$$B_i^* B_i = E_i^* \varepsilon_{ij} E_j . \tag{24}$$

Using Eq. (24) in Eq. (23), we have

$$\langle W \rangle = \frac{1}{16\pi} E_i^* E_j \left[\varepsilon_{ij} + \frac{\partial}{\partial \omega} (\omega \varepsilon_{ij}) \right] = \frac{E_i^* E_j}{16\pi\omega} \frac{\partial(\omega^2 \varepsilon_{ij})}{\partial \omega}. \tag{25}$$

We will now rewrite Eq. (25) in terms of Λ, the determinant of the operator Λ_{ij}. To do this we note that

$$\Lambda_{ij} E_j \equiv \lfloor n^2 (k_{ij} - \delta_{ij}) + \varepsilon_{ij} \rfloor E_j = 0,$$

or

$$n^2 (\delta_{ij} - k_{ij}) E_j = \varepsilon_{ij} E_j ,$$

with

$$k_{ij} \equiv k_i k_j / k^2, \tag{26}$$

where $n \equiv ck/\omega$ is the plasma index of refraction.

Consider now

$$\frac{\partial}{\partial \omega} (\omega \Lambda_{ij}) = \Lambda_{ij} + \omega \frac{\partial \Lambda_{ij}}{\partial \omega} = \varepsilon_{ij} + n^2 (k_{ij} - \delta_{ij}) + \omega \frac{\partial \Lambda_{ij}}{\partial \omega}$$

$$= \varepsilon_{ij} + n^2 (k_{ij} - \delta_{ij}) + \omega \frac{\partial \varepsilon_{ij}}{\partial \omega} - 2n^2 (k_{ij} - \delta_{ij})$$

$$= n^2 (\delta_{ij} - k_{ij}) + \frac{\partial(\omega \varepsilon_{ij})}{\partial \omega} \tag{27}$$

since

$$\frac{\partial}{\partial \omega} n^2 = -2n^2/\omega.$$

It then follows from Eq. (27) that

$$E_i^* E_j \frac{\partial}{\partial \omega} (\omega \Lambda_{ij}) = E_i^* \left\{ \frac{\partial}{\partial \omega} (\omega \varepsilon_{ij}) + n^2 (\delta_{ij} - k_{ij}) \right\} E_j$$

$$= E_i^* E_j \frac{1}{\omega} \frac{\partial}{\partial \omega} (\omega^2 \varepsilon_{ij}), \tag{28}$$

where we have used Eq. (26).

Combining Eqs. (25) and (28), we have

$$\langle W \rangle = \frac{1}{16\pi} E_i^* E_j \frac{1}{\omega} \frac{\partial}{\partial \omega} (\omega^2 \varepsilon_{ij})$$

$$= \frac{1}{16\pi} E_i^* E_j \frac{\partial}{\partial \omega} (\omega \Lambda_{ij}) = \frac{1}{16\pi} |E|^2 \frac{\lambda_{ij}}{\lambda_0} \frac{\partial}{\partial \omega} (\omega \Lambda_{ij}), \tag{29}$$

where we have used the relation

$$\lambda_{ij} = \lambda_0 e_i^* e_j = \lambda_0 E_i^* E_j |E|^{-2}$$

in the last step. In the above formula λ_0 is the trace of the cofactor matrix λ_{ij} of Maxwell's operator Λ_{ij}, and e_i is the polarization vector. Now

$$\lambda_{ij} \frac{\partial}{\partial \omega} (\omega \Lambda_{ij}) = \lambda_{ij} \left[\Lambda_{ij} + \omega \frac{\partial \Lambda_{ij}}{\partial \omega} \right]$$

$$= \lambda_{ij} \Lambda_{ij} + \omega \lambda_{ij} \frac{\partial \Lambda_{ij}}{\partial \omega} = 3\Lambda + \omega \frac{\partial \Lambda}{\partial \omega} = \omega \frac{\partial \Lambda}{\partial \omega}, \tag{30}$$

since for any real mode the determinate of Maxwell's operator (written in matrix notation) vanishes, i.e., $\Lambda = 0$, and

$$\frac{\partial \Lambda}{\partial \omega} = \lambda_{ij} \frac{\partial}{\partial \omega} \Lambda_{ij} ,$$

which can be easily shown by direct differentiation of

$$\Lambda \equiv \det \Lambda_{ij} = \Lambda_{ij} \lambda_{ij}$$

with respect to ω. From Eqs. (29) and (30) we finally obtain

$$\langle W \rangle = \frac{|\vec{E}|^2}{16\pi} \frac{\lambda_{ij}}{\lambda_0} \left[\Lambda_{ij} + \omega \frac{\partial \Lambda_{ij}}{\partial \omega} \right]$$

$$= \frac{|\vec{E}|^2}{16\pi} \frac{\omega}{\lambda_0} \frac{\partial \Lambda}{\partial \omega} = \frac{1}{8\pi} (\mathrm{Re}\, \vec{E})^2 \frac{\omega}{\lambda_0} \frac{\partial \Lambda}{\partial \omega}, \tag{31}$$

which is the desired result.

III. THE VECTOR POTENTIAL

We are now ready to derive an expression for the vector potential which is valid in a magnetoactive plasma. Such an expression is extremely useful for astrophysical applications, because some of the most important astrophysical quantities such as the radiative opacities and luminosities can be calculated most conveniently in terms of the vector potential. A knowledge of the radiative opacities is absolutely essential for the description of the energy transfer in stellar interiors, and, on the other hand, in order to calculate the surface temperature and cooling rates of stars as well as stellar evolution, one needs to known the stellar energy-loss rates due to the radiation of photons and neutrinos. Since most of these radiation processes occur in a magnetized plasma, it is obvious that a general expression for the vector potential which takes into account plasma effects is of great interest.

Turning now to the mathematical details, let us represent the electromagnetic field in a magnetized plasma by the four-potential $A_\mu(\vec{A},i\phi)$, and choose the gauge such that $\phi=0$. The vector potential $\vec{A}(\vec{x},t)$, which is real, can be expanded in the form

$$\vec{A}(\vec{x},t)=\sum_{\vec{k},\alpha=1,2}\{\vec{U}_{\vec{K},\alpha}(\vec{x})e^{-i\omega t}a_{k,\alpha}+\vec{U}^{*}_{\vec{k},\alpha}(\vec{x})e^{i\omega t}a^{\dagger}_{k,\alpha}\} \tag{32}$$

$$\vec{U}_{\vec{k},\alpha}(\vec{x})=|\vec{U}_{\vec{k},\alpha}|e^{i\vec{k}\cdot\vec{x}}\hat{e}_{k,\alpha},$$

where $a_{k,\alpha}$ and $a^{\dagger}_{k,\alpha}$ are the annihilation and creation operators for a plasmon with wave vector $\vec{k}$ and direction of polarization along the unit vector $\hat{e}_k$; α denotes such a direction. The coefficient $|\vec{U}_{k,\alpha}|$ is then determined by quantizing the electromagnetic fields. To do this, we note that for n plasmons per unit volume, the average energy density $\langle W_\alpha\rangle$ of the radiation field is given by Eq. (31),

$$\langle W_\alpha\rangle=\frac{1}{8\pi}\langle|\mathrm{Re}\,\vec{E}_\alpha|^2\rangle\frac{\omega}{\lambda_0}\frac{\partial}{\partial\omega}\Lambda,$$

$$=n_\alpha\hbar\omega \tag{33}$$

where the electric field is obtained from the vector potential

$$\vec{E}=-\frac{1}{c}\frac{\partial\vec{A}}{\partial t}. \tag{34}$$

From Eqs. (32), (33), and (34) we have

$$\vec{A}_{k,\alpha}(\vec{x},t)=2|\vec{U}_{\vec{k},\alpha}|\hat{e}_{k,\alpha}\cos(\vec{k}\cdot\vec{x}-\omega t)$$

$$\langle\dot{W}_\alpha\rangle=\frac{1}{8\pi}\left\langle\frac{1}{c^2}\left[\frac{\partial}{\partial t}(\mathrm{Re}\,\vec{A})\right]^2\right\rangle_\tau\frac{\omega}{\lambda_0}\frac{\partial\Lambda}{\partial\omega}$$

$$=\frac{1}{8\pi c^2}\langle[2|\vec{U}_{\vec{k},\alpha}|\omega]^2\sin^2(\vec{k}\cdot\vec{x}-\omega t)\rangle_\tau\frac{\omega}{\lambda_0}\frac{\partial\Lambda}{\partial\omega}$$

$$=\frac{1}{8\pi c^2}\left[4\omega^2\left|\vec{U}_{\vec{k},\alpha}\right|^2\left(\frac{1}{2}\right)\right]\frac{\omega}{\lambda_0}\frac{\partial\Lambda}{\partial\omega}$$

$$=n_\alpha\hbar\omega=\frac{N_\alpha}{\Omega}\hbar\omega, \tag{35}$$

which yields

$$|\vec{U}_{\vec{k},\alpha}|=\left(\frac{2\pi\hbar^2c^2N_\alpha}{(\hbar\omega)\Omega}\right)^{1/2}\left(\frac{2\lambda_0}{\omega\frac{\partial\Lambda}{\partial\omega}}\right)^{1/2}. \tag{36}$$

In Eq. (36), Ω is the normalization volume in which there are N_α plasmons (i.e., $n_\alpha=N_\alpha/\Omega$). The subscript τ in Eq. (35) indicates that the time average is taken over one period so that

$$\langle\sin^2(\vec{k}\cdot\vec{x}-\omega t)\rangle_\tau=\tfrac{1}{2}.$$

The plane-wave expansion of the vector potential is therefore

$$\vec{A}(\vec{x},t)=\sum_{\vec{k}}\sum_{\alpha=1,2}N^{1/2}_\omega\{e^{i(\vec{k}\cdot\vec{x}-\omega t)}a_{\vec{k},\alpha}+e^{-i(\vec{k}\cdot\vec{x}-\omega t)}a^{+}_{\vec{k},\alpha}\}\hat{e}_{\vec{k},\alpha},$$

$$N_\omega=\left(\frac{2\pi\hbar c^2}{\Omega\omega}\right)\left(\frac{2\lambda_0}{\omega\frac{\partial\Lambda}{\partial\omega}}\right)=\frac{2\pi\hbar^2c^2}{E_\omega\omega}\zeta(\omega), \tag{37}$$

$$E_\omega=\hbar\omega,\quad \omega=\omega(k),$$

where we have assumed that there is one plasmon in the volume Ω so that $n_\alpha=(N_\alpha/\Omega)=(1/\Omega)$. We note that the factor $\zeta(\omega)$ in Eq. (37) represents the effect of the plasma whereas the factor $2\pi(\hbar c)^2/E_\omega\Omega$ is simply the vacuum limit for the vector potential.

To illustrate the usefulness of Eq. (37), let us compute $\zeta(\omega)$ for transverse waves in a nonmagnetized electron plasma for which the Maxwell matrix Λ_{ij} and its associated cofactor matrix λ_{ij} may be written as

$$\Lambda_{ij}=\begin{bmatrix}P-n^2\cos^2\theta & 0 & n^2\sin\theta\cos\theta\\ 0 & P-n^2 & 0\\ n^2\sin\theta\cos\theta & 0 & P-n^2\sin^2\theta\end{bmatrix}, \tag{38}$$

$$\lambda_{ij}=\begin{bmatrix}(P-n^2)(P-n^2\sin^2\theta) & 0 & (P-n^2)n^2\sin\theta\cos\theta\\ 0 & P(P-n^2) & 0\\ (P-n^2)n^2\sin\theta\cos\theta & 0 & (P-n^2)(P-n^2\cos^2\theta)\end{bmatrix}, \tag{39}$$

where θ is the angle between the wave vector $\vec{k}$ and the Z axis, with $k_y=0$, i.e., $(n=ck/\omega, P=1-(\omega_P^2/\omega^2))$,

$$\vec{k}=(k_x,0,k_z)=k(\sin\theta,0,\cos\theta).$$

From Eqs. (38) and (39) the factor $\zeta(\omega)$ may be evaluated as follows:

$$\zeta(\omega)=2\frac{\lambda_0}{\omega\frac{\partial\Lambda}{\partial\omega}}=\frac{2T_r(\lambda_{ij})}{\omega\frac{\partial}{\partial\omega}\|\Lambda_{ij}\|}$$

or

$$\frac{\omega}{2}\zeta(\omega)=\frac{(P-n^2)(3P-n^2)}{\frac{\partial}{\partial\omega}[P(P-n^2)^2]}\Bigg|_{n^2=P}=\frac{1}{\frac{\partial P}{\partial\omega}+\frac{2P}{\omega}}.$$

Hence

$$\zeta(\omega)=\frac{2}{\omega}\frac{1}{\frac{2P}{\omega}+\frac{\partial P}{\partial\omega}}=\frac{2}{2P+\omega\frac{\partial P}{\partial\omega}}$$
$$=\frac{2}{2n^2+\omega\frac{\partial}{\partial\omega}n^2}=\frac{1}{n\frac{\partial}{\partial\omega}(\omega n)}, \tag{40}$$

where we have used L'Hospital's rule and the dispersion relation $n^2=P$ for transverse plasmons.

Similarly, for longitudinal plasmons it can be easily shown that

$$\lambda_0^l=\mathrm{Tr}\ \lambda_{ij}^l=1,\quad \Lambda\equiv\det(\Lambda_{ij}^l)=P,$$

where the superscript l indicates longitudinal waves. In this case $\hat{e}=\hat{k}$ and the dispersion relation is simply $P=0$ or $\omega=\omega_p$. The factor $\zeta(\omega)$ may now be evaluated to give

$$\zeta(\omega)=\frac{2\lambda_0^l}{\omega\frac{\partial\Lambda}{\partial\omega}}=\frac{2}{\omega\frac{\partial P}{\partial\omega}}=\frac{\omega^2}{\omega_p^2} \tag{41}$$

or

$$\zeta(\omega)=1.$$

Equations (40) and (41) are used by Adams, Ruderman, and Woo [21] for computing the neutrino energy-loss rates by plasmon neutrino processes, which are important for late stage stellar evolution. The general expression $\zeta(\omega)$ is also applicable in a magnetoactive plasma, for instance Canuto *et al.* [15] have considered the photoneutrino process in a strong magnetic field and the same process in a magnetoactive electron plasma has been investigated by Chou, Fassio-Canuto, and Canuto [20].

We note that in the absence of the plasma, $n=ck/\omega=1$ and Eq. (40) gives $\zeta(\omega)=1$ so that

$$N_\omega=\frac{2\pi\hbar^2c^2}{E_\omega\Omega}\zeta(\omega)=\frac{2\pi\hbar c^2}{\Omega\omega},$$

which is simply the vacuum limit for the vector potential. In other words, we recover the electromagnetic waves in vacuum.

IV. THE COLLISIONLESS COLD PLASMA MODEL

The possible modes of excitation for a strongly magnetized electron plasma have been investigated by several authors [12,29] and discussed by Canuto, Chiuderi, and Chou [18,19] in connection with the plasmon neutrino process. This excitation frequency has also been used to compute the photoneutrino luminosity for transverse plasmons by Chou, Fassio-Canuto, and Canuto [20]. To simplify our analysis, we make few approximations to obtain explicit functional forms for the excitation frequencies. We neglect both spatial dispersion due to electron pressure for an electron gas of arbitrary degeneracy and the effects of spin and collisions on the dielectric tensor. The effect of spatial dispersion has been shown by Adams, Ruderman, and Woo [21] to be unimportant in the high-density regime such as the interiors of neutron stars and dwarf stars, whereas the spin effects on the dielectric tensors are negligible [30]. In a superstrong magnetoactive electron plasma, the cyclotron frequency is so high that particle correlation is due to magnetic fields rather than collisions: hence the effects of collisions on the dielectric tensor can also be neglected.

We will now compute the normalization factor ζ previously derived for the collisionless cold plasma model. It is anticipated that the normalization factor will appear in the quantum-mechanical treatment for the nonlinear interaction between intense neutrino fluxes and a magnetoactive electron plasma background. In view of the approximations delineated above for the plasma immersed in superstrong magnetic fields, we shall therefore proceed to compute the normalization factor for a cold collisionless plasma model. In the notation of Stix [31], the dielectric tensor can be written

$$\varepsilon_{ij}=\begin{bmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{bmatrix},$$

$$S=\tfrac{1}{2}(R+L), \quad D=\tfrac{1}{2}(R-L), \quad P=1-\frac{\omega_p^2}{\omega^2}, \quad \omega_c=\frac{eB}{mc}, \tag{42}$$

$$R=1-\frac{\omega_p^2}{\omega(\omega-\omega_c)}, \quad L=1-\frac{\omega_p^2}{\omega(\omega+\omega_c)}, \quad \omega_p^2=\frac{4\pi Ne^2}{m},$$

where N is the electron number density and ω_p is the plasma frequency. The index of refraction satisfies the biquadratic equation

$$An^4-Bn^2+C=0,$$

$$A\equiv S\sin^2\theta+P\cos^2\theta, \tag{43}$$

$$B=RL\sin^2\theta+SP(1+\cos^2\theta),$$

$$C=RPL.$$

The solutions are given by

$$n^2=\frac{B\pm F}{2A}, \quad F^2=B^2-4AC.$$

Equation (43) can be cast in a more transparent form [32,33],

$$\tan^2\theta=-\frac{P(n^2-R)(n^2-L)}{(Sn^2-RL)(n^2-P)}, \tag{44}$$

which shows that for propagation along the magnetic field $\theta=0$ we have either $n_0^2=R$ for the ordinary mode O or $n_x^2=L$ for the extraordinary mode X. Similarly, for propagation across the magnetic field ($\theta=\pi/2$) we have the ordinary mode $n_0^2=P$ and the extraordinary mode $n_x^2=RL/S$. The latter, being a linearly polarized mixed mode, is partially transverse and partially longitudinal. It can be easily shown that $\mathrm{Tr}(\lambda_{ij})$ is given by

$$\mathrm{Tr}(\lambda_{ij})=n^4-(P+A+2S)n^2+RL+2PS.$$

The general form of the polarization vector e has been considered by Melrose, whose result is

$$e_i=\frac{\lambda_{ij}C_j}{[C_i^*\lambda_{ij}C_j\mathrm{Tr}(\lambda_{ij})]^{1/2}}.$$

The index l specifying the various modes of excitation is implicitly contained in λ_{ij} through the index of refraction n_l. The constant parameters C_j are a set of complex numbers with the constraint that one of them should not vanish. Let us choose

$$\vec{C}=(0,i,0).$$

We then obtain after some straightforward computation

$$e_i=[1+G_l^2]^{-1/2}\left\{\frac{D(P-n_l^2\sin^2\theta)}{SP-An_l^2},i,\frac{-Dn_l^2\sin\theta\cos\theta}{SP-An_l^2}\right\}, \tag{45}$$

$$G_l=\frac{PD\cos\theta}{SP-An_l^2}.$$

It follows that

$$\left|\frac{\mathrm{Tr}(\lambda_{ij})}{\partial\|\Lambda_{ij}\|/\partial n^2}\right|_{\|\Lambda_{ij}\|=0}=\left|\frac{n^4-(P+A+2S)n^2+(RL+2SP)}{\pm F}\right|, \tag{46}$$

since $2An^2-B=\pm F$.

Equation (46) is the desired general result for the normalization factor $\zeta(\omega)$ for a cold collisionless magnetoactive electron plasma. As previously emphasized, this factor is expected to appear in a full-fledged quantum-mechanical description for the nonlinear interaction between intense neutrino fluxes and stellar plasma background immersed in a superstrong magnetic field.

ACKNOWLEDGMENTS

The author gratefully acknowledges the financial support from the National Science Council of the Republic of China under Grant No. NSC-88-2112-M-008-022. The author would also like to thank Dr. L. H. Li of the Purple Mountain Observatory for useful discussions.

[1] L. Woltjer, Astrophys. J. **140**, 1309 (1964).
[2] H. J. Lee, V. Canuto, H. Y. Chiu, and C. Chiuderi, Phys. Rev. Lett. **23**, 390 (1969).
[3] D. H. Brownell and J. Callaway, Nuovo Cimento B **60b**, 169 (1969).
[4] S. D. Silverstein, Phys. Rev. Lett. **23**, 139 (1969).
[5] E. Ostgaard, Nucl. Phys. A **154**, 202 (1970).
[6] C. M. Lewis, Phys. Rev. D **44**, 1661 (1991).
[7] E. H. Harrison, MNRAS **147**, 279 (1970).
[8] R. Opher and U. F. Wichoski, Phys. Rev. Lett. **78**, 787 (1997).
[9] P. K. Shukla, L. Stenflo, R. Bingham, H. A. Bethe, J. M. Dawson, and J. T. Mendonca, Phys. Lett. A **233**, 181 (1997).
[10] M. A. Ruderman, Phys. Rev. Lett. **27**, 1306 (1971).
[11] L. Spruch, R. Mueller, and A. R. Rau, Phys. Rev. Lett. **26**, 1136 (1971).
[12] V. Canuto and C. K. Chou, Phys. Fluids **16**, 1273 (1973).

[13] V. Canuto and H. Y. Chiu, Phys. Rev. **173**, 1210 (1968); **173**, 1229 (1968)
[14] V. Canuto and L. Fassio-Canuto, Phys. Rev. D **7**, 1593 (1973).
[15] V. Canuto, and L. Fassio-Canuto, Phys. Rev. D **7**, 1601 (1973).
[16] V. Canuto, C. Chiuderi, C. K. Chou, and L. Fassio-Canuto, Astrophys. Space Sci. **28**, 145 (1974).
[17] Z. G. Dai, Tan. Lu, and Qiu-He Peng, Sci. China, Ser. A **36**, 1358 (1993).
[18] V. Canuto, C. Chiuderi, and C. K. Chou, Astrophys. Space Sci. **7**, 407 (1970).
[19] V. Canuto, C. Chiuderi, and C. K. Chou, Astrophys. Space Sci. **9**, 453 (1970).
[20] C. K. Chou, L. Fassio-Canuto, and V. Canuto, Astrophys. J. **227**, 974 (1979).
[21] J. B. Adams, M. A. Ruderman, and C. H. Woo, Phys. Rev. **129**, 1383 (1963).
[22] D. B. Melrose, Astrophys. Space Sci. **2**, 171 (1968).
[23] T. Chiueh, Astrophys. J. **413**, L35 (1993).
[24] R. Bingham, J. M. Dawson, J. J. Su, and H. A. Bethe, Phys. Lett. A **193**, 279 (1994).
[25] R. Bingham, H. A. Bethe, J. M. Dawson, P. K. Shukla, and J. J. Su, Phys. Lett. A **220**, 107 (1996).
[26] P. K. Shukla, R. Bingham, H. A. Bethe, J. M. Dawson, and L. Stenflo, Phys. Scr. **55**, 96 (1997).
[27] P. K. Shukla, L. Stenflo, R. Bingham, L. A. Bethe, J. M. Dawson, and J. T. Mendonca, Phys. Lett. A **224**, 239 (1997).
[28] H. A. Bethe, Phys. Rev. Lett. **56**, 1305 (1986).
[29] Y. Klimontovich and V. P. Silin, in *Plasma Physics*, edited by J. E. Drummond (McGraw-Hill, New York, 1961), Chap. 2
[30] P. Burt and H. Wahlquist, Phys. Rev. **125**, 1785 (1962).
[31] T. H. Stix, *The Theory of Plasma Waves* (McGraw-Hill, New York, 1962).
[32] E. O. Astrom, Ark. Fys. **2**, 443 (1950).
[33] W. P. Allis, S. J. Buchsbaum, and A. Bers, *Waves in Anisotropic Plasmas* (MIT Press, Cambridge, MA, 1963).